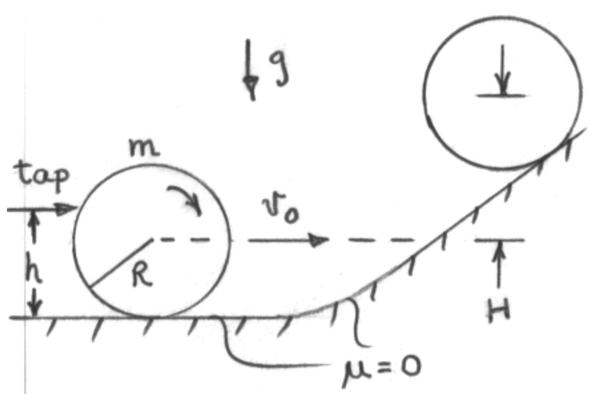
University of California, Berkeley
Physics H7A Fall 1998 (*Strovink*)

SOLUTION TO EXAMINATION 2

1.



(a.) The horizontal tap produces a (horizontal) *linear* impulse $\int F\,dt \equiv J$. With respect to the center of the cylinder, it also produces a (clockwise) *angular* impulse $\int \tau\,dt = Jb$, where $b = h - R$ is the impact parameter of the horizontal tap. Then, in terms of J, since the cylinder is initially at rest,

$$mv_0 = J$$
$$I\omega_0 = J(h - R)$$

Substituting $I = \frac{1}{2}mR^2$, and imposing the condition $v_0 = R\omega_0$ that the cylinder rolls without slipping, these equations become

$$mv_0 = J$$
$$\frac{1}{2}mR^2\frac{v_0}{R} = J(h - R)$$

These equations are mutually consistent only if $h - R = R/2$, or

$$h = \frac{3}{2}R$$

(b.) Since the ice is frictionless, and the force of gravity acts on the C.M. of the cylinder, no torques about its axis can be exerted on the cylinder. Therefore its angular momentum and angular velocity $\omega = \omega_0$ remain constant even on the hill. Considering that the kinetic energy of a rigid body decomposes into $K_{\text{trans}} + K_{\text{rot}}$, we conclude that only $K_{\text{trans}} = \frac{1}{2}mv_0^2$ is available to be converted into potential energy mgH. Therefore the maximum height is

$$H = \frac{v_0^2}{2g}$$

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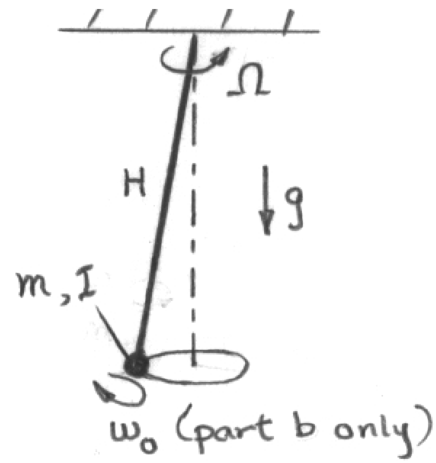
2.



(a.) Let β be the angle between the stick and the vertical. This part can easily be worked in the C.M. of the bead, where a centrifugal force $m\Omega^2 r = m\Omega^2 H \sin\beta$ balances the horizontal component $mg\tan\beta$ of the tension $mg/\cos\beta$ in the stick. Or it can be worked in the lab, where the horizontal component of the tension supplies the necessary centripetal acceleration. Or, elegantly, the circular motion of the bead can be considered to be the superposition of an x pendulum and, delayed by one-quarter of a period, a y pendulum with the same amplitude. When the approximation $\beta \ll 1$, valid for part (a.), is applied, β cancels out, and any of these approaches yields the usual result

$$\Omega = \sqrt{\frac{g}{H}}$$

(b.) Take the origin to be the pivot point. If the bead is spinning with constant angular velocity ω_0 about the stick's axis, the vertical component

$I\omega_0 \cos\beta$ of the spin angular momentum L remains constant. But the horizontal component $L_h = I\omega_0 \sin\beta$ of L precesses with angular velocity Ω, as in a gyroscope. The torque ΩL_h that is required to maintain this precession is the torque due to gravity, $mgr = mgH \sin\beta$; the stick can't supply this torque because its end coincides with the origin. Then

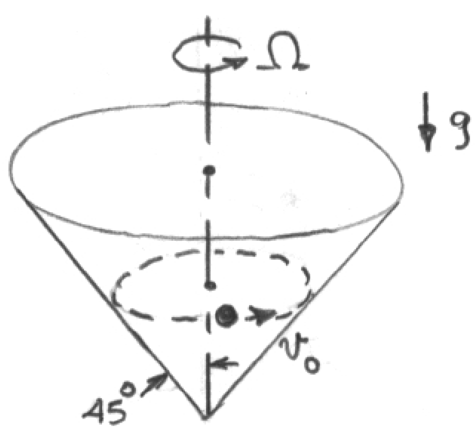
$$\Omega I\omega_0 \sin\beta = mgH \sin\beta$$

The angular velocity of precession is

$$\Omega = \frac{mgH}{I\omega_0}$$

independent of β. Because $\sin\beta$ cancels out of the previous equation, there is no restriction on it and therefore no restriction on the orbit radius $r = H\sin\beta$; this circular motion can occur for any orbit radius $r \leq H$ provided that that ω_0 is large enough to allow the spin angular momentum to dominate the other angular momenta in the problem.

3.



(a.) This part of the problem can be done by balancing forces; the normal force of the 45° cone on the pebble has equal horizontal and vertical components mg. When the horizontal component is equated to the centripetal acceleration $mv_0\Omega$, we obtain immediately

$$\Omega = \frac{g}{v_0}$$

Anticipating what will be needed for part (b.), we can also solve part (a.) using the effective potential

$$U_{\text{eff}} = \frac{l^2}{2mr^2} + mgr$$

where r is the perpendicular distance of the pebble to the cone axis. (In the second term, we are using the fact that, for a 45° cone with $z = r$, an increase Δr causes an increase $mg\Delta z = mg\Delta r$ in the true potential energy.) Then a circular orbit occurs when

$$0 = \frac{dU_{\text{eff}}}{dr} = -\frac{2l^2}{2mr^3} + mg$$

Substituting $l^2 = mv_0 r \times m\Omega r^2$ causes the r dependence to cancel:

$$\begin{aligned} 0 &= -\frac{2m^2 v_0 \Omega}{2m} + mg \\ \Omega &= \frac{g}{v_0} \end{aligned}$$

as before.

(b.) Proceeding with the effective potential method, we obtain the effective spring constant k_{eff} for radial motion by differentiating U_{eff} again with respect to r:

$$\begin{aligned} k_{\text{eff}} &= \frac{d}{dr}\Big(-\frac{l^2}{mr^3} + mg\Big) \\ &= \frac{3l^2}{mr^4} \\ &= \frac{3m^2 r^4 \Omega^2}{mr^4} \\ &= 3m\Omega^2 \end{aligned}$$

Thus

$$\omega_r = \sqrt{\frac{k_{\text{eff}}}{m}} = \sqrt{3}\Omega = \sqrt{3}\frac{g}{v_0}$$

(c.) Since the ratio of ω_r to Ω is irrational, an integer number of orbital cycles cannot occur in the same time interval as an integer number of radial cycles. Therefore the orbit never repeats itself and so is not closed.